THE DEBRUIJN ALGORITHM FOR QUASICRYSTALS AS A MODEL FOR NON-LOCAL SYMMETRY

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Abstract: *Nicolaas Govert deBruijn's (1918-2012) algorithm does its quasicrystal-generation magic by bouncing between two states of a six-dimensional cubic lattice: the unprojected state and a projected state. In a physical quasicrystal the bridge/barrier between these two states is an entropy event.*
Author's Note: *I remember a lecture, somewhere, during which Roger Penrose suggested that since no foolproof local matching rules exist for quasicrystals, real, physical quasicrystals must assemble via a global, quantumlike method. This paper follows that conjecture with the specifics of the deBruijn algorithm.*

Keywords: Quasicrystals, non-Local phenomena, deBruijn algorithm.

INTRODUCTION

Famously, Einstein, Podolsky, and Rosen (EPR, 1935) complained that if quantum theory is true and complete then there would be absurd action at a distance. Eighty-five years later, we imagine **information** to be a physical entity, one intrinsically bound to entropy, and EPR's complaint might now be restated as: if quantum theory is true and complete then there would be absurd information at a distance.

Entangled particles (what EPR predicted) are created as a pair with an inevitable simple symmetry. Then they are released into the world to build separate histories; they become asymmetrical. Upon later examination of the particles, they seem to have recreated their earlier, simpler symmetry. If an object (or system) can no longer be described by a set of data but only by a larger set of data, then the object has passed through an entropy event, meaning that the object or system should not be able to revert to the simpler, original set of data; that violates the law of entropy. But entangled particles seem to do just that: they seem to retain a simple, global identity as they fortify a more complex local identity. Symmetry-asymmetry-resymmetry. The deBruijn algorithm interchangeably uses two sets of data describing the same system, and that is why it can be a useful model for non-locality.

1 TWO AXIOMS

Projection is a one-way gate. The amount of information necessary to describe a system is changed by projection; projection is an entropy event; and thus, it creates an arrow of time. If a three-dimensional geometric structure is projected to two dimensions, then the information about its position in three-dimensional space (and the position of its parts) is immediately lost, probably never to be reconstructed. If that mathematical structure represents a physical structure, an arrow of time is created. Despite that loss, the volume of information is <u>increased</u> by projection. Here is the *FormianK* code to generate a 6x6x6x6x6x6 cubic lattice:

lattice=pex|rinic(1,2,3,4,5,6,6,6,6,6,6,6,1,1,1,1,1,1)|pan(6,0)|pan(5,0)|pan(4,0)|rosad(0.5,0.5)|{[0,0,0;1,0,0],[0,0,1;1,0,1],[0,0,0;0,0,1]};

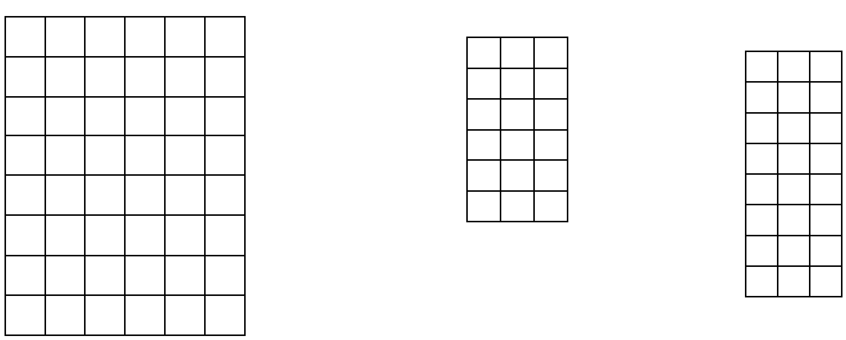
My *Pascal* code to generate a portion of this structure projected to three dimensions so as to generate a mathematical quasicrystal is 626 lines long, linked here: http://tonyrobbin.net/pdfs/MCALC.PAS
Let's call this changed volume of information "the projected state."

Translated structures provide information at a distance. Visualize the 6x6x6x6x6x6 cubic lattice just mentioned to be in front of you. Rotate the lattice so that you are looking down the fifth-dimensional row at the bottom of the stacks. Tell us about the fourth cube in the fifth-dimensional row. You know its size, shape, orientation, and how it is connected to its neighbors. (These are the things that are so mysterious about a cell in a quasicrystal.) You know this standing at the origin. You know this because that far cube is just like the cube before you. Let's call this "the unprojected state."

2 THE DEBRUIJN ALGORITHM

At the end of the algorithm there is a matrix multiplication of 6x8 (A) and 3x6 (B) multiplied on the left resulting in a matrix 3x8 (C). Matrix A is a three-dimensional cube - eventually a cell in the quasicrystal - that is plucked from the six-dimensional lattice in its unprojected state; it has 8 vertices, each having 6 coordinates. Matrix B is the projection matrix given in the deBruijn algorithm. Matrix B is composed of the unit vectors normal to the faces of a dodecahedron; it is what happens to the 6 mutually perpendicular axes of the origin when projected, as well as what happens to all the nodes of the lattice when projected in this way. Matrix B is often called a "star of vectors." Matrix C is the resulting projected cell; it has 8 vertices but only 3 coordinates, the other 3 having been lost by projection. Repeated application of the algorithm to other cells in the unprojected lattice fills in the three-dimensional quasicrystal.

Matrix A　　Matrix B　　Matrix C　　**C = BA**



Below is a photo (Photo 1) of models of the matrix multiplication. The projection results in one of only two cell shapes:

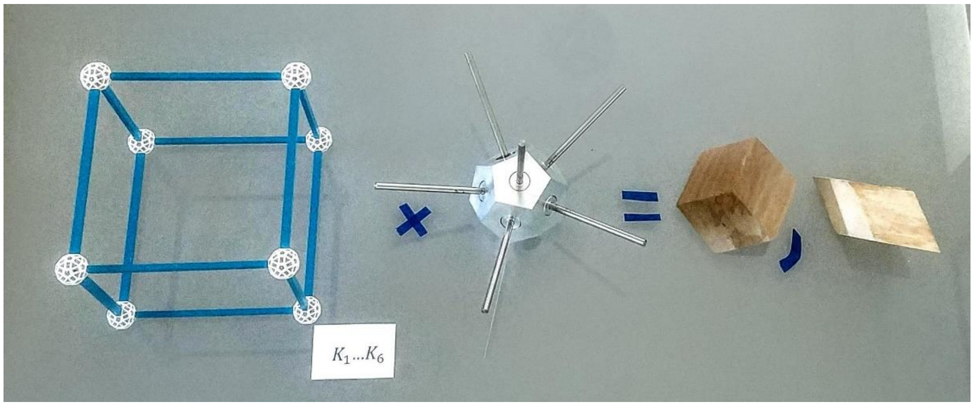
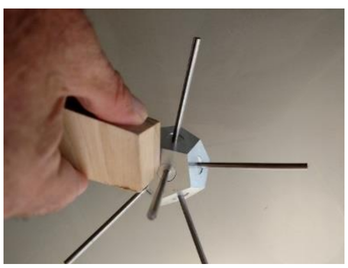


Photo 1

Movie 1 clearly shows why choosing vectors in the star three at a time (for a 3d quasicrystal[i]) can only result in one of these two cells, in one of twenty discrete orientations: 6 choose 3 = 20 combinations.



Movie 1 linked here: http://tonyrobbin.net/pdfs/Movie1.mp4

What is not clear is <u>where</u> such a projected cell would be in three-dimensional space: how far and in what direction the cell would be from the origin. The placement of this quasicrystal cell in 3-space is a function of where it was plucked from in 6-space, but that is precisely the information lost by the projection.

3 RECOVERY OF INFORMATION LOST IN PROJECTION

DeBruijn's algorithm is based on two marvelous visualizations: first, that quasicrystals can be generated by projecting a higher-dimensional cubic lattice in just the right way, and second, that information lost by projection can be recovered with a clever technique. Planes are constructed normal to the star of vectors, as in Photo 2 and Drawing 1, below. Every two planes will intersect in a line. A third such plane will intersect that line, identifying a unique point in 3-space. (Three planes normal to three perpendicular axes will have a point in common, and this remains true as those axes are projected; it is just that the intersection point will slide as the system is projected away from its original rectilinear arrangement.) It takes a bit of thinking to realize that this intersection point will never be part of the final quasicrystal. In conversation, deBruijn emphasized the abstract nature of this intersection point by saying it has only a "topological" meaning. Nevertheless, the vector from the origin to this topological point (another intermediate, fictitious element) when projected to the <u>other</u> three axes can identify the ordinal locations on the star of vectors of a vertex of a cube in 6-space. These are ordinal points; their cardinal values are identified only by reference to the original unprojected lattice. The other seven vertices can be specified by successively advancing along the vectors. When that is done, the three-dimensional cell, which now has a specified location in 6-space, can be projected one more time to give its location in the final quasicrystal.

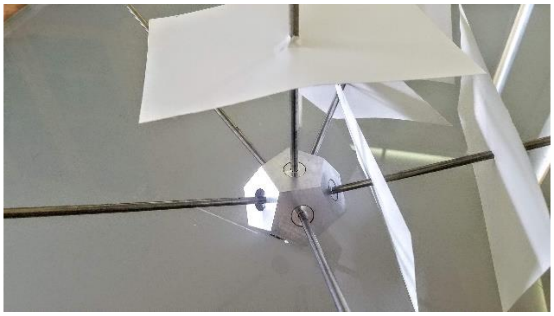
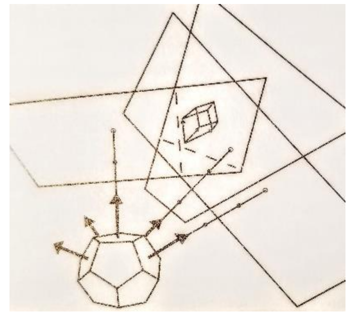


Photo 2



Drawing 1

4 IF PHYSICAL QUASICRYSTALS ARE LIKE THE DEBRUIJN ALGORITHM, WHAT JUST HAPPENED?

The steps in the deBruijn algorithm are these: First, the entire 6-dimensional cubic lattice is projected to 3- space. This is a one-way gate. Next, work is done on that projection: 3 of the 6 rays of the star of vectors are chosen at random. (All combinations of rays will eventually be cycled through.) Planes are constructed normal to those three rays, and three of those planes are simultaneously solved to find a unique intersection point. Still in 3-space, a vector to that point is projected to the remaining three rays of the star of vectors to find the ordinal location on each remaining ray. Now, the algorithm reverses, goes back through the one-way gate the wrong way to do work on the original, unprojected six-dimensional cubic lattice. A vertex of that unprojected lattice has been specified by work just done in three dimensions, and from that vertex a cell in 6-space is discovered: it is now a three-dimensional cell that has a location in 6-space. That location is no longer merely “topological,” but instead has metric values. This is the cell defined in Matrix A, which is projected back to 3-space, passing again through the one-way gate the right way, to its position in the final quasicrystal. At last, the poor overworked cell has a shape, an orientation, and a position in the final three-dimensional quasicrystal.

The algorithm uses the projected lattice and the unprojected lattice interchangeably; if it is physics instead of math, the algorithm bounced back and forth between two sets of information about the lattice without regard for the entropic implications. Said another way, the process holds two states of matter in superposition until a resolution by a final “observation” (an interaction) is made. Or, long distance information from the unprojected lattice, the translated lattice, is available to work being done on the other side of an entropy event. This is a geometric model of non-locality.

APPENDIX 1

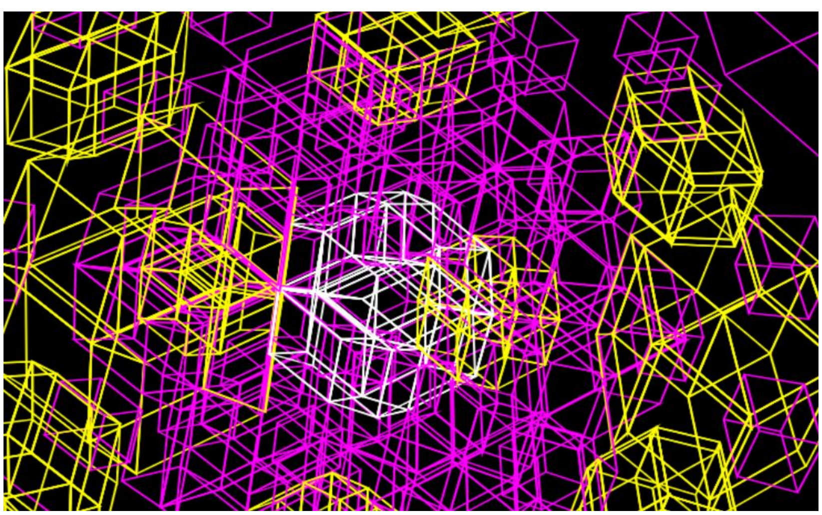
To make the individual cells of a quasicrystal without reference to the star of vectors see Movie 2. This movie provides additional insights into the fat and skinny cells. http://tonyrobbin.net/pdfs/Movie2.mp4

APPENDIX 2

The cells discussed in the text also are part of sub-assemblies of the quasicrystal. The boundary of a cube is not its edges but the planes of the cube. In general, the boundary or hull of an n-dimensional object is made of n-1 dimensional objects that are its components. Thus a six-dimensional cube is made up of, and bounded by, five-dimensional cubes that are, in turn, made up, and bounded by, four-dimensional cubes, that are made up of, and bounded by, the three-dimensional cubes discussed in the text. The deBruijn algorithm recovers these nested figures as the four zonohedra: projected six-dimensional cubes become rhombic triacontahedra; five-dimensional cubes become rhombic icosahedra; four-dimensional cubes become rhombic dodecahedra; all made up of fat and skinny cells.

APPENDIX 3

A high-resolution database of 2,500 cells is available in the arXiv, https://arxiv.org/abs/1805.11457 Readers are invited to enhance the database by translating it into AutoCad, or Rhino, or VectorWorks, etc.. Please share the translations with me. Also linked in that arXiv article is an improved version of my quasicrystal program by George Francis and his students that will run on just about any computer, smartphone, or tablet. It shows the quasicrystal with its component zonohedra, and can be rotated at will. See below:



The Francis program can also be linked directly here: http://new.math.uiuc.edu/Tavares/htr4.html

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[i] The great mystery of the deBruijn algorithm is how the section of cells to include is made: why are only cells that tessellate in 3-space in the final projection? Why are there no interlacing cells, and why are there no holes in the lattice? Afterall, a shadow of a skeletal cube shows the top and bottom of the cube together, interlaced on the flat surface, and a projection of the entire six-dimensional cubic lattice into 3-space has massive interlacing. In a way, the "selection" is made by the user of the algorithm who has decided to take all combinations of the projected axes three at a time; then, all results proceed as a matter of course. I realize that this answer is anthropic and elusive, but perhaps it is sufficient. By choosing three axes at a time to work with, the user is guaranteed a three-dimensional quasicrystal. By adding unit vectors from the star of vectors to build a cell from a discovered vertex, the user is guaranteed a consistent set of unit cells. By taking the planes-normal one after the other, the user is guaranteed a continuous tessellation. (Now here comes the hard part.) By choosing to start each search for a cell with all combinations of all six of the axes taken three at a time, the user is guaranteed a closed packing.

Gilboa, 2020, during pandemic lockdown.